Teaching briefs... Investigating Pythagorean Triples

by Ken Sullins

For the past two years I've begun my class *Finite/Discrete Math: An Introduction* with an investigation of the four sets of Pythagorean Triples on the right.

The students are given the four sets and are allowed to investigate the sets for a couple of days on their own and in small groups, with minimal directions and hints from me. I want them to have the freedom to ask their questions, make their observations, make their conjectures, and make their connections as they investigate the individual or groups of sets.

We then discuss Set 1 in class and I write the summary of our conjectures, observations, discoveries, and extensions for the next class discussion.

The next day we discuss Set 2 and write a report together in class. This writing has taken one or two days. It has been a very interesting experience for the class; trying to organize so many thoughts into a collaborative report is difficult, but provides a great experience. Remember, this is probably their first experience in writing a report in mathematics--their experiences in writing science lab reports are helpful, but the writing of mathematical concepts and explanations is likely new.

Once we have completed the class report in rough form, they select either Set 3 or Set 4 for their report. My basic rule for working together is that students can discuss anything from beginning to end (this freedom is another new experience for most of them), but when something needs to be written (in a report or on a take home quiz) it should be done after the discussion is completed and be their own writing. Time and patience by both student and teacher are important here.

The one question that always comes up in one way or another is "How do I know when I'm done?" My answer is either "When you've got no more questions to ask" or "When you're satisfied there's nothing else to find." The students come to realize that you really only take a break from the problem--you keep coming back over a period of time to look for something else!

I believe these are the types of investigations the *Standards* support. Give these sets to your classes (or those on page 7), turn them loose and see what happens! Let their questions and answers, investigations and discoveries, and conjectures, take them to a different level of problem solving.

Here are excerpts from the class report for Set 1.

We first considered the number in column **a**. These are the odd numbers and we wanted a general expression for them. To represent any even number we can use the expression $2n$, where n is any natural number; so to get the odds we can use the expression $2n - 1$.

We then looked for something relating the rows of numbers, which are Pythagorean triples. It was noted that the numbers in each row were related by $a^2 + b^2 = c^2$ and that the numbers in columns **b** and **c** were consecutive integers. Dave also noticed that if you add the numbers from the columns **a** and **b** in one row to the column **a** number in the next row you get the number in column **b** of that second row!

We then looked at a new problem solving approach, graphing, to see what type of relation might exist between the term number (we created a new column) and the numbers in the columns. The column **a** numbers were graphed (n, a), where n was the term number. We could clearly see that this did give us a line; that line was what we had found earlier---$a = 2n - 1$. For the first time students were exposed to the connection between discrete (dots) and continuous (line) graphs.

The first thing we noticed for column **b** was that the numbers are multiples of 4; Mr. Sullins told us that the second factors were called triangular numbers and explained

SET ONE

1	0	1
3	4	5
5	12	13
7	--	25
9	40	--
11	--	--
--	--	85

SET TWO

4	3	5
8	15	17
12	35	37
16	--	---
20	--	101
--	--	---

SET THREE

12	5	13
20	21	29
28	--	53
36	77	--
44	--	--
--	--	--

SET FOUR

9	0	9
15	8	17
21	20	29
27	36	--
--	--	65
39	--	--
--	--	--

Complete the above tables. Discover the patterns. Find relationships among each triple. Seek anything else within each set.